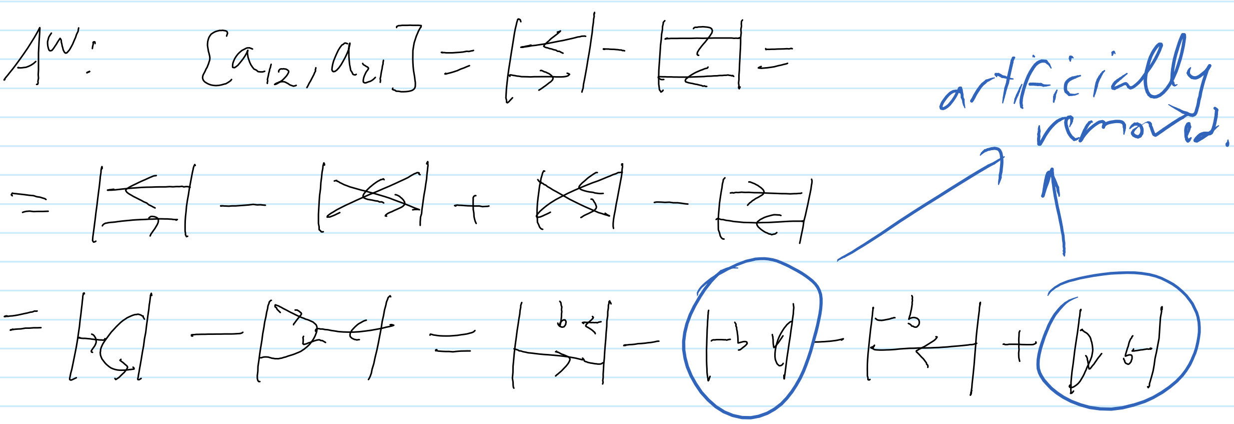
$[a_{ij},a_{ji}]$

April-14-15 11:47 AM

(150412) Deriving Gassner: $[a_{ij},a_{ik}]=0$, $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, $[a_{ij},a_{ji}]=?$, b_i central. Acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\operatorname{ad}a_{ij}}x_i=x_i$, $e^{\operatorname{ad}a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$. Renaming $y_i=x_i/b_i$, $t_i=e^{b_i}$, get $[e^{\operatorname{ad}a_{ij}}]_{y_i,y_j}=\begin{pmatrix}1 & 1-t_i\\ 0 & t_i\end{pmatrix}$.

In $\mathcal{A}^w$: $[a_{12},a_{21}]=$



$s[a_{ij},a_{ji}]=a_{ij}b_j-a_{ji}b_i$ How fix sign?

$[[a_{ij},a_{ji}],a_{ik}]\overset{?}{=}[[a_{ij},a_{ik}],a_{ji}]+[a_{ij},[a_{ji},a_{ik}]]$

$s[a_{ij}b_j-a_{ji}b_i,a_{ik}]\overset{?}{=}0+[a_{ij},b_ja_{ik}-b_ia_{jk}]$

$sb_i(b_ia_{jk}-b_ja_{ik})\overset{?}{=}b_j(b_ja_{ik}-b_ia_{jk})$

So $s=-1$: $[a_{ij},a_{ji}]=b_ia_{ji}-b_ja_{ij}$